A General SPICE Model for Arbitrary Linear Dispersive Multiport Components Described by Frequency-Domain Data

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Abstract — **The paper introduces a general and efficient approach to dispersive multiport component modeling for use in conjunction with SPICE-like time-domain transient analysis programs. The model computes at any time instant the port currents making use of the time-dependent exciting voltages and of frequency-domain data generated once for all by electromagnetic or circuit analysis.**

I. Introduction

In the transient simulation of nonlinear integrated circuits, signals with spectra ranging well into the GHz region must be handled as a matter of course. This is not only true for microwave/mm-wave analog circuits, but for many digital circuits as well: for instance, the clock frequency of state-of-the-art microprocessors presently exceeds 2 GHz. In such broad frequency ranges, passive multiport components such as directional couplers, corporate feed networks, and multiwire interconnects, normally exhibit a strongly dispersive behavior. As a consequence, the natural approach to the characterization of such devices is frequency-domain electromagnetic (EM) simulation, which is now well established. In particular, there is barely an alternative to this choice when the layout shape makes it impossible to describe the device as the interconnection of elementary components, and/or strong EM couplings exist between different parts of the circuit. On the other hand, the time-domain characterization of the same class of devices for use in conjunction with SPICE-like transient simulators, is still an open problem. Lumped element equivalent circuits are cumbersome to derive, and are often inaccurate when the number of ports exceeds 2. For some specific multiports such as multiwire interconnects, highly specialized and sophisticated modeling techniques have been developed [1], [2], but for generic devices reliable models of universal use do not seem to be available.

The paper proposes a general-purpose numerical SPICE model, which for our present purposes is defined as a subprogram that computes the port currents (the component response) at a generic time instant starting from the knowledge of the time-dependent port voltages (the excitation). The device description consists exclusively of its frequency-domain scattering or admittance parameters, which may be generated by any frequency-domain analysis procedure, including layout-based EM simulation. This information is used in a preprocessing step to derive the ramp responses of the transfer functions between all couples of ports. The exciting voltages are then approximated by piecewise linear functions, and the transient response is evaluated with high numerical efficiency. The model has been implemented into SPICE [4] as a user-defined device, and has been successfully used in the transient and steady-state time-domain analysis of several kinds of circuits containing passive multiport dispersive components.

In this summary, for validation purposes the model is used in conjunction with SPICE to compute in the time domain the transient and steady-state responses of typical multiwire interconnects that can also be analysed by the telegraphers' equations approximation [1] - [3]. The transient results are in excellent agreement with those produced by well-known specialized algorithms [1] - [3], with the significant advantage that with our model the CPU time increases very slowly as a function of circuit complexity. The steady-state results are strictly identical to those obtained by standard Fourier analysis. More extensive results including EM-based applications will be provided in the final version of the paper.

II. Description of the Model

Let us consider a linear N-port network $\mathcal{N}$ of arbitrary topology excited by a set of voltages $V_j(t)$ applied to its ports ($1 \leq j \leq N$). We shall assume that a frequency-domain description of $\mathcal{N}$ is available in terms of its frequency-dependent N x N admittance matrix $\mathbf{Y}(\omega)$ where ω ranges from zero up to some frequency Ω larger than the maximum frequency of interest for circuit analysis purposes. The choice of Ω will be further discussed later on. A generic entry of $\mathbf{Y}(\omega)$ will be denoted by $Y_{ij}(\omega)$. The time-domain response of $\mathcal{N}$ to the set of excitations $V_j(t)$ will be defined as the set of time-dependent currents $I_j(t)$ entering the network ports. The purpose of this section is to compute such time-domain response under the following assumptions:

1) the time variable is uniformly discretized, so that the response need only be computed at a set of sampling instants (SI)

$$t_n = t_1 + (n-1)\,\tau \qquad (1)$$

where n is an arbitrary integer.

2) for $t \leq t_1$ the circuit is in stationary (DC) conditions, or equivalently the excitations satisfy the conditions

$$V_j(t) = V_j(t_1) \qquad (t \leq t_1,\ 1 \leq j \leq N) \qquad (2)$$

Note that no assumptions of any kind are made on the frequency dependence of $\mathbf{Y}(\omega)$, nor on the time dependence of the excitations $V_j(t)$. In order to solve the problem in an accurate and efficient way, we first introduce the unit ramp function $r(t)$ of duration τ, defined by

$$r(t,\tau) = \begin{cases} 0 & \text{if } t \leq 0 \\ \dfrac{t}{\tau} & \text{if } 0 \leq t \leq \tau \\ 1 & \text{if } t \geq \tau \end{cases} \qquad (3)$$

When a voltage excitation expressed by $r(t, \tau)$ is applied to the j-th port and all the remaining ports are short-circuited, the time-domain current entering the i-th port will be denoted by $R_{ij}(t, \tau)$. Let us now consider a generic voltage excitation $g(t)$ applied to the j-th port. $g(t)$ is assumed to satisfy a constraint similar to (2). Since only the response values at the SI are of interest, in the time interval $t_1 \leq t \leq t_n$ $g(t)$ may be replaced by the piecewise linear approximation

$$g(t) \approx g(t_1) + r(t - t_1, \tau)\cdot[g(t_2) - g(t_1)] + $$
$$+ r(t - t_2, \tau)\,[g(t_3) - g(t_2)] + \ldots\ldots = $$
$$= g(t_1) + \sum_{p=2}^{n} \left[g(t_p) - g(t_{p-1})\right] r(t - t_p, \tau) \qquad (4)$$

as shown in fig. 1. The summation in (4) is understood to be zero for $n = 1$. For any real network $R_{ij}(t, \tau)$ must be causal ($R_{ij}(t, \tau) = 0$ for $t < 0$). At the i-th port, the augmented network response to the excitation (4) will thus be a current of the form

$$I_{ij}(t) = Y_{ij}(0)\, g(t_1) + \sum_{p=2}^{n} \left[g(t_p) - g(t_{p-1})\right] R_{ij}(t - t_p, \tau) \qquad (5)$$

where $Y_{ij}(0)$ is evaluated at DC. The n-th time-domain sample of $I_{ij}(t)$ is then

$$I_{ij}(t_n) = Y_{ij}(0)\, g(t_1) + \sum_{p=2}^{n} \left[g(t_p) - g(t_{p-1})\right] R_{ij}(t_n - t_p, \tau) \qquad (6)$$

In the general case, the network excitation consists of all the voltages $V_j(t)$ applied to the ports. The time-domain sample of the i-th current at the n-th SI then takes on the expression

$$I_i(t_n) \approx \sum_{j=1}^{N} Y_{ij}(0)\, V_j(t_1) + + \sum_{j=1}^{N} \sum_{p=2}^{n} \left[V_j(t_p) - V_j(t_{p-1})\right] R_{ij}(t_n - t_p, \tau) \qquad (7)$$

(7) allows the circuit response to be efficiently computed at any time after the samples of the ramp responses $R_{ij}(t, \tau)$ have been computed and stored in the preprocessing step.

III. Computation of the Ramp Responses

For maximum computational efficiency, the ramp responses $R_{ij}(t, \tau)$ are computed by ordinary frequency-domain analysis starting from trapezoidal excitations with rising and falling edges of duration τ. The sampled values of all the $R_{ij}(t, \tau)$ at all the SI are found and stored prior to the beginning of circuit analysis. In this way, the analysis based on (7), despite being conceptually equivalent to a time-domain convolution, is much better conditioned and much faster.

Let us consider the periodic trapezoidal excitation $g(t)$ of period $2M\tau$ defined by

$$g(t) = \sum_{p=-\infty}^{\infty} \{r(t - 2pM\tau, \tau) - r[t - (2p+1)M\tau, \tau]\} \qquad (8)$$

The fundamental angular frequency of $g(t)$ is obviously

$$\omega_G = \frac{\pi}{M\tau} \qquad (9)$$

$g(t)$ is uniformly sampled in the time domain with step $\tau/2^c$ where c is a positive integer greater than 0. The FFT is then used to compute the harmonics G_h of g(t). The number of available harmonics is given by

$$H = \mathrm{int}\left[\frac{2^{c+1}M - 1}{2}\right] \tag{10}$$

so that the following Fourier expansion holds for g(t):

$$g(t) = \sum_{h=-H}^{H} G_h \exp(jh\omega_G t) \tag{11}$$

If g(t) is now applied as a voltage excitation to the j-th network port, the current response $\rho_{ij}(t)$ at the i-th port is obviously

$$\rho_{ij}(t) = \sum_{h=-H}^{H} Y_{ij}(h\omega_G)\, G_h \exp(jh\omega_G t) \tag{12}$$

so that the sampled values of $\rho_{ij}(t)$ may be evaluated once again by the FFT after computing once for all the harmonics G_h. Note that (12) sets to $\Omega = H\omega_G$ the upper bound of the frequency range where the admittance matrix $\mathbf{Y}(\omega)$ must be known. $R_{ij}(t)$ may then be generated from $\rho_{ij}(t)$ by letting

$$R_{ij}(t) = \begin{cases} 0 & \text{for } t < 0 \\ \rho_{ij}(t) & \text{for } 0 \le t \le M\tau \\ Y_{ij}(0) & \text{for } t > M\tau \end{cases} \tag{13}$$

(13) takes advantage of the fact that the asymptotic value of $R_{ij}(t)$ for $t \to \infty$ is given by $Y_{ij}(0)$, and is thus a priori known. Of course, the value of M will have to be set in such a way that acceptable convergence to such asymptotic value be achieved for $t = M\tau$.

For maximum accuracy, the sampled values of the responses $R_{ij}(t)$ at all the SI should be stored in the computer memory. This is always done if sufficient memory is available. On the contrary, if the storage requirement exceeds the available memory, the program first computes the maximum number of samples, say S, that can be stored. The ramp response sampling interval is then redefined as

$$\tau_S = 2^{Q-c}\,\tau \tag{13}$$

where Q is an integer (Q > c). Q is iteratively increased, while holding the total number of samples equal to S, until the condition

$$\left|R_{ij}[(S-r)\tau_S] - Y_{ij}(0)\right| < \varepsilon \tag{14}$$

is satisfied for $0 \le r \le 5$, where ε is a predefined threshold. Finally, only the samples

$$R_{ij}(s\tau_S) \quad (1 \le s \le S) \tag{15}$$

are stored in the computer memory, while all intermediate values required for the computation of (6) are found by linear interpolation.

IV. Applications

As a first application, we consider the distributed interconnect network shown in fig. 2, which has been used by several authors as a standard benchmark (e.g., [3]). The network Y matrix is computed in the frequency domain, and the samples of the ramp responses $R_{ij}(t)$ are found and stored. This preprocessing step takes about 5 seconds of CPU time and requires 2.3 MB of storage on an 800 MHz PC. As an example of the results generated in this way, fig. 3 shows the ramp responses at ports 1, 3, normalized to $Y_{11}(0)$, $Y_{13}(0)$, respectively, and fig. 4 shows the ramp response at port 4 (in this case $Y_{12}(0) = 0$). A periodic trapezoidal excitation with rise/fall times of 0.1 ns and pulse width of 2.9 ns is then applied to the input port. In fig 5 the voltages at ports 1 - 3 generated by SPICE in conjunction with our model are compared with the results derived in [3] for the same circuit, and are found to be virtually coincident. The CPU time required for the generation of these plots is about 126 seconds. In fig. 6 one period of the steady-state waveforms reached by SPICE after 100 cycles of a periodic trapezoidal excitation with rise/fall times of 0.2 ns and pulse width of 4.8 ns is compared with the results generated by Fourier analysis: in this case the two sets of results are strictly identical. As a further example, fig 8 shows the transient responses generated by SPICE by means of our model at ports 1 - 3 of the 4-section interconnect depicted in fig. 7. The point here is that the CPU time required to generate fig. 8 is about 119 seconds, which is more or less the same as that for fig. 5, in spite of a four-fold increase in circuit complexity.

References

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[3] S. Pasha, M. Celik, A.C. Cangelleris and J.L. Prince, "Passive Spice-compatible models of Dispersive Interconnects", *IEEE Electronic Components and Technology Conference*, 1999, pp. 493-499.

[4] SpectreHDL: *Cadence Design Systems.*

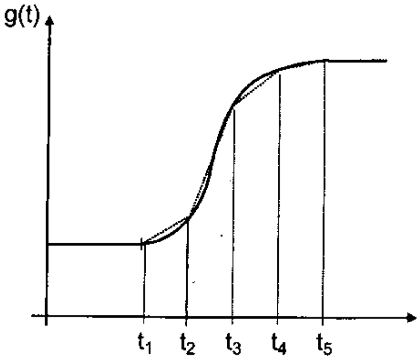

Fig. 1 Time domain piecewise linear approximation of a generic voltage excitation

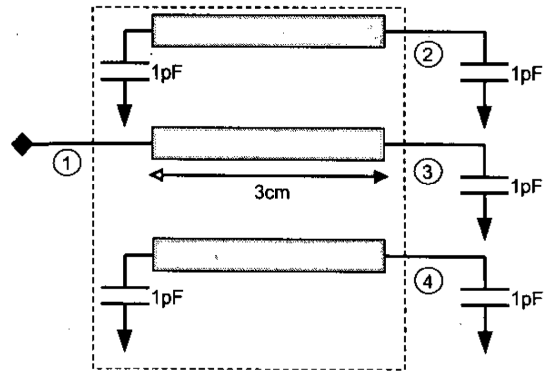

Fig. 2 Interconnect network with three coupled lines

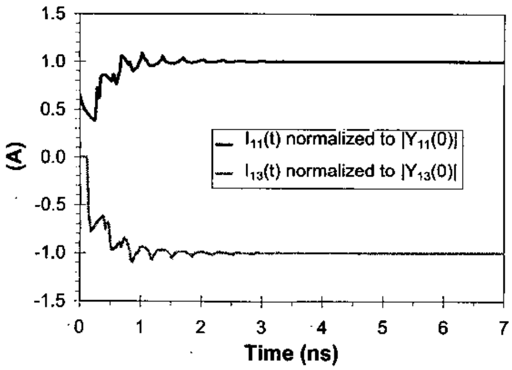

Fig. 3 Network response at port 1 and 3 to a ramp excitation g(t) applied to port 1

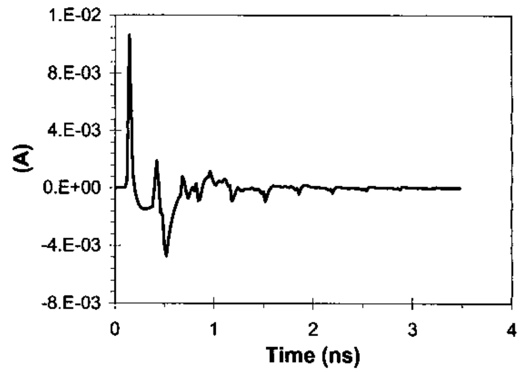

Fig. 4 Network response at port 2 to a ramp excitation g(t) applied to port 1

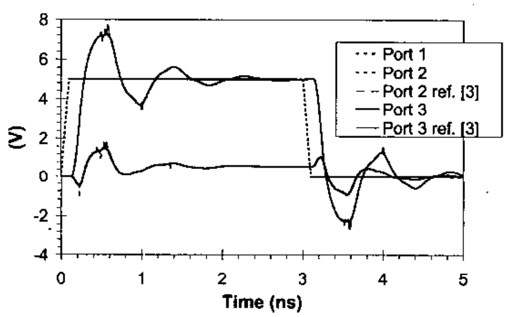

Fig. 5 Output waveforms obtained by Spice in conjunction with our model

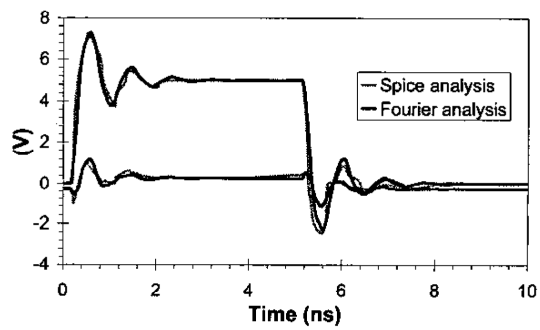

Fig. 6 Steady-state output waveforms

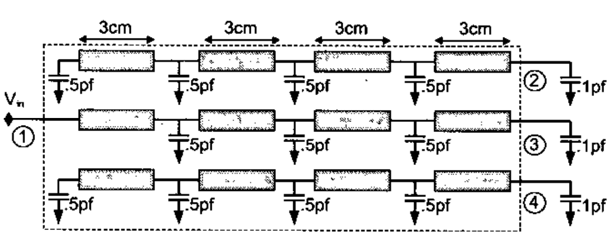

Fig. 7 4-section coupled line interconnect

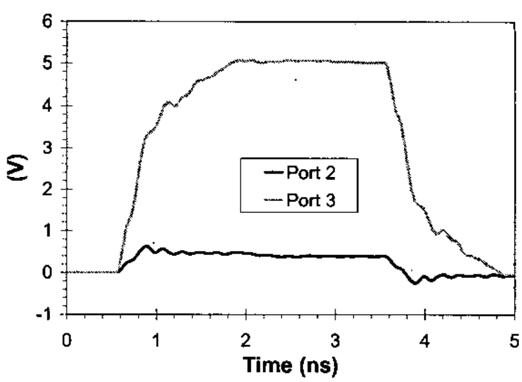

Fig. 8 Output waveforms of the 4-section interconnect